48. The top and the bottom each need one 9-inch strip and one 14-inch strip. The four sides each need one 5-inch strip. So, the total length needed is $2(9 + 14) + 4(5) = 66$ inches.

49. Draw a number line on which each tick mark represents 3 feet. Label the line to show each boy’s position. The completed diagram shows that Paul was 3 feet ahead of George.

50. The dashed segments in the following diagram represent the ladder in various positions as it slides down the wall. The midpoint of each segment is marked. The path traced by the midpoint is an arc of a circle or a quarter-circle if the ladder slides all the way from the vertical to the horizontal.

51. He will get home at 5:46 (assuming he goes inside before he gets blown back again).

52. If the triangle is rotated 90° clockwise, $\overline{AB}$ will be a vertical segment. Point $B$ will have the same location as it does now, and point $A$ will be at $(2, 3)$.

53.  

54.  

55.  

56.  

57.  

58.  

59.  

60.  

61. "All rocks sink." Story needs to find one rock that will not sink.

62. Possible conjecture: If two angles are formed by drawing a ray from a line, then their measures add up to 180°.

63. 10,000, 100,000. Each term is 10 times the previous term.

64. $\frac{5}{6}$, 1. Written with the common denominator 6, the sequence becomes $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}, \ldots$.

65. $-17, -21$. Four is subtracted from each term to get the next term.

66. 28, 36. To get from term to term, you add 2, then add 3, then add 4, and so on.

67. 21, 34. To find each term, you add the two previous terms.

68. 49, 64. The terms are the squares of consecutive whole numbers: $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, \ldots$. The next two terms are $7^2 = 49$ and $8^2 = 64$.

69. $-10, -24$. To get from term to term, you subtract 2, then subtract 4, then subtract 6, and so on.

70. 64, 128. Each term is double the previous term.

71. Each figure has one more point than the previous figure. Each point in a figure is connected to each of the other points.

72. To get the next figure, increase the number of rows by 2 and the number of columns by 1.

73. To get the next figure, add one row to the bottom and one column to the right of the previous figure, and then shade all the rectangles in the bottom row but the rightmost one.
4. To create each figure, add two branches to each of the new branches from the previous figure.

5. To create each figure, connect the midpoints of the sides of each shaded triangle of the previous figure, dividing it into four triangles, and then color the middle triangles white.

6. The nth figure is an n-by-n grid of squares with a shaded circle inscribed in each square.

7. The first term is \(3(1) - 2\), the second term is \(3(2) - 2\), the third term is \(3(3) - 2\), and so on. So, the first five terms are 1, 4, 7, 10, 13.

8. The fifth term is \(\frac{5(5+1)}{2}\), the sixth term is \(\frac{6(6+1)}{2}\), and so on. So, the next five terms are 15, 21, 28, 36, 45.

9. Sample answer: 1, -2, 4, -8, 16, . . . Each term is -2 times the previous term.

10. Sample answers: 3, 6, 12, 24, 48, . . . and 4, 8, 12, 16, 20, . . .

11. Sample answer: I learned by trial and error that you turn wood screws clockwise to screw them into wood and counterclockwise to remove them. This is inductive reasoning because it involves making generalizations from patterns.

12. 7th term: 56; 10th term: 110; 25th term: 650. Look for a pattern in the arrangements of dots. Notice that in each figure, the number of columns of dots is one greater than the number of rows. Also notice that the number of rows is the same as the term number. Thus, the nth figure has \(n\) rows and \((n + 1)\) columns, so it contains \(n(n + 1)\) dots. To find the 7th term, let \(n = 7\). \(7(7 + 1) = 7(8) = 56\). Likewise, the 10th term is \(10(11) = 110\), and the 25th term is \(25(26) = 650\).

13. The conjecture is false. Sample counterexample: \(14^2 = 196\) but \(41^2 = 1681\).

14. In each case, the middle digit in the product is the number of 1's in each factor. The digits to the left of the middle digit are consecutive integers, from 1 up to the middle digit. The digits to the right of the middle digit are the same as the digits on the left, but in reverse order. So

\[
11,111 \cdot 11,111 = 123,454,321
\]

\[
111,111 \cdot 111,111 = 12,345,654,321
\]

However, in the tenth line, 1,111,111,111 is multiplied by itself. The digits can't go up to 10, so, carrying the 1, you get

\[
1,111,111,111 \cdot 1,111,111,111 = 1,234,567,900,987,654,321
\]

15. Turn your book so that the red line is vertical. Imagine rotating the figure so that you can see faces on both sides of the red line. Possible answers:

16. Sample answer:

17. Sample answer: collinear

18. Sample answer: dodecagon

19. Sample answer: protractor

20. Sample answer: diagonal

21. Sample answer: 90°

22. Sample answer:

23. Sample answer:
43. Sample answer:

44. Possible answer:

45. Possible answer: Draw $\overline{AB}$ with length 9 cm. Using $A$ as the vertex, draw a 40° angle with $\overline{AB}$ along one side. Then, using $B$ as the vertex, draw a 60° angle that also has $\overline{AB}$ along one side. Extend the sides of the two angles until they intersect. Their point of intersection, $C$, will be the third vertex of the triangle.

No. It is not possible to draw a second different triangle with the same two angle measures and side length between them. A second triangle drawn with these given measures would be congruent to the first one.

**IMPROVING YOUR REASONING SKILLS**

1. 9. The sequence is the perfect squares, in reverse order, written backward. That is, the first term is 9², or 81, written backward; the second term is 8², or 64, written backward; and so on. Because the last term listed is 4², or 16, written backward, the next term must be 3², or 9, written backward. But 9 written backward is just 9, so the next term is 9.

2. T. The terms are the first letters in the words *One*, *Two*, *Three*, *Four*, and so on. The last term listed is the first letter in *Nine*, so the next term must be *T*, the first letter in *Ten*.

3. 64. The sequence can be rewritten as 1, 2², 3, 4², 5, 6², 7. The next term must be 8², or 64.

4. 8671. To find each term, you double the previous term and then write the digits in the reverse order. To find the next term, double 884 to get 1768 and then reverse the digits to get 8671.

5. 18. The numbers added to or subtracted from each term to get the next term are consecutive primes. The pattern alternates between addition and subtraction. The next term is 1 + 17, or 18.

6. 6. Each number is the number of “open ends” on the previous letter. The letter G has two open ends.

7. 2. The steps below show how the sequence is generated. To get the third term, multiply the first two terms, 2 and 3, and write the product, 6, at the end of the list. To generate the next term, multiply the 3 (the second term you used to find the previous product) and the next term, 6. Write the digits of the product, 1 and 8, as separate terms at the end of the list. To get the next term, multiply the 6 (the second term you used to find the previous product) by the next term, 1, and write the product’s digits (in this case, just 6) at the end of the list. Continue this process to get the remaining terms.

2, 2 → 2 · 3 = 6 → 2, 3, 6
2, 3, 6 → 3 · 6 = 18 → 2, 3, 6, 1, 8
2, 3, 6, 1, 8 → 6 · 1 = 6 → 2, 3, 6, 1, 8, 6
2, 3, 6, 1, 8, 6 → 1 · 8 = 8 → 2, 3, 6, 1, 8, 6, 8
2, 3, 6, 1, 8, 6, 8 → 8 · 6 = 48 → 2, 3, 6, 1, 8, 6, 8, 4, 8
2, 3, 6, 1, 8, 6, 8, 4, 8, 4 → 6 · 8 = 48 → 2, 3, 6, 1, 8, 6, 8, 4, 8, 4, 8
2, 3, 6, 1, 8, 6, 8, 4, 8, 4, 8, 4 → 8 · 4 = 32 → 2, 3, 6, 1, 8, 6, 8, 4, 8, 4, 8, 4, 8, 3, 2
2, 3, 6, 1, 8, 6, 8, 4, 8, 4, 8, 3, 2 → 4 · 8 = 32 → 2, 3, 6, 1, 8, 6, 8, 4, 8, 4, 8, 3, 2, 3, 2
2, 3, 6, 1, 8, 6, 8, 4, 8, 4, 8, 3, 2, 3, 2 → 8 · 4 = 32 → 2, 3, 6, 1, 8, 6, 8, 4, 8, 4, 8, 3, 2, 3, 2

8. The letters in the top row are formed from straight segments only. The letters in the bottom row have curves. Because X, Y, and Z have only straight segments, they belong in the top row.

**EXTENSION**

Discussions will vary.

**LESSON 2.2**

**EXERCISES**

1. $6n - 3; 117$. Possible method: The difference between terms is always 6, so the rule is $6n + "something."$ Let $c$ stand for the unknown "something." The first term, $f(1)$, is 3, so $6(1) + c = 3$. Solving this equation gives $c = -3$. So, the rule is $f(n) = 6n - 3$. To find the 20th term, substitute 20 for $n$: $f(20) = 6(20) - 3 = 120 - 3 = 117$.

2. $-3n + 4; -56$. Possible method: The difference between the terms is always $-3$, so the rule is $-3n + c$ for some number $c$. To find $c$, use the fact that the value of the first term, $f(1)$, is 1. So, $-3(1) + c = 1$. Solving this equation gives $c = 4$, so the rule is $f(n) = -3n + 4$. To find the 20th term, substitute 20 for $n$: $f(20) = -3(20) + 4 = -56$.

3. $8n - 12; 148$
5. $8n$: 1600. Possible method: Fill in the table for the first four terms. The difference is always 8. Continue this pattern for terms 5 and 6. The rule is $8n + c$ for some number $c$. Because the first term is 8, $8(1) + c = 8$, and so $c = 0$. Thus, the rule is $8n$. The number of tiles in the 200th figure must be $8(200)$, or 1600.

<table>
<thead>
<tr>
<th>Figure number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>$n$</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of tiles</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
<td>40</td>
<td>48</td>
<td>$8n$</td>
<td>1600</td>
</tr>
</tbody>
</table>

6. $4n - 3$; 797. Possible method: Fill in the table for the first four terms. The difference is always 4. Continue this pattern for terms 5 and 6. The rule is $4n + c$ for some number $c$. Because the first term is 1, $4(1) + c = 1$, and so $c = -3$. Thus, the rule is $4n - 3$. The number of tiles in the 200th figure must be $4(200) - 3$, or 797.

<table>
<thead>
<tr>
<th>Figure number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>$n$</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of tiles</td>
<td>1</td>
<td>5</td>
<td>9</td>
<td>13</td>
<td>17</td>
<td>21</td>
<td>$4n - 3$</td>
<td>797</td>
</tr>
</tbody>
</table>

7. Number of matchsticks: $4n + 1$; 801. Possible method: Fill in the table for the first four terms. The difference of the numbers in the "number of matchsticks" row is always 4. Use the pattern to fill in the numbers for figures 5 and 6. The rule is in the form $4n + c$. Using the first term, $4(1) + c = 5$. Therefore, $c = 1$, so the rule for the number of matchsticks is $4n + 1$. The 200th figure has $4(200) + 1$, or 801, matchsticks.

Matchsticks in perimeter: $3n + 2$; 602. Possible method: The difference of the numbers in the "number of matchsticks in perimeter of figure" row is always 3. Use this fact to fill in the numbers for figures 5 and 6. The rule is in the form $3n + c$. Using the first term, $3(1) + c = 5$. Therefore, $c = -2$, so the rule for the number of matchsticks in the perimeter is $3n + 2$. The 200th figure has $3(200) + 2$, or 602, matchsticks in its perimeter.

<table>
<thead>
<tr>
<th>Figure number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>$n$</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of matchsticks</td>
<td>5</td>
<td>9</td>
<td>13</td>
<td>17</td>
<td>21</td>
<td>25</td>
<td>$4n + 1$</td>
<td>801</td>
</tr>
<tr>
<td>Number of matchsticks in perimeter of figure</td>
<td>5</td>
<td>8</td>
<td>11</td>
<td>14</td>
<td>17</td>
<td>20</td>
<td>$3n + 2$</td>
<td>602</td>
</tr>
</tbody>
</table>

8. The points for $8n$ (Exercise 5) lie on the steepest line. The coefficient of $n$ gives a measure of the steepness. (It is the slope of the line.)

9. $C_{2n+2}$. Each carbon atom has one hydrogen atom above it and one hydrogen below it, plus there is always one hydrogen atom on each end of the chain. So, if there are $n$ carbon atoms, there are $2n + 2$ hydrogen atoms.

\[
\begin{align*}
H & \quad H \quad H \quad H \quad H \quad H \quad H \\
& \quad C \quad C \quad C \quad C \quad C \quad C \quad C - C - C - C \\
& \quad H \quad H \quad H \quad H \quad H \quad H \quad H \\
\end{align*}
\]

Octane ($C_{10}H_{22}$).

10. $y = \frac{3}{2}x + 3$. Possible reasoning: The slope of the line through the points is $\frac{9 - 3}{4 - 2}$, or $\frac{3}{2}$, and the $y$-intercept is 3. So the equation is $y = \frac{3}{2}x + 3$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-3</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

11.

12.

13.

14.

15. Possible answer: Márisol should point out that although all the triangles José drew were isosceles, it is possible to draw a triangle with no two sides congruent. She should then show José such a counterexample.

16. Possible answer: Use the ruler to draw $AB$ of length 8 cm. Use the protractor to draw a 45° angle with point $A$ as its vertex. Use the ruler to measure a distance 9 cm along the side of the angle that does not lie along $AB$ to form $AC$. Connect
points $B$ and $C$ to form the third side of triangle $ABC$.

![Triangle diagram]

No. A second triangle drawn with these given measures would be congruent to the first one.

17. She could try eating one food at a time; inductive.

18. 2600. Look for a pattern in the arrangements of squares in the rectangular arrays. Notice that in each one, the number of columns of squares is two greater than the number of rows. Also notice that the number of rows is the same as the array number. Thus, the $n$th array has $n$ rows and $(n + 2)$ columns, so it contains $n(n + 2)$ squares. To find the number of squares in the 50th array, let $n = 50$: $50(52) = 2600$, so the 50th array contains 2600 squares.

**PROJECT**

Project should satisfy the following criteria:

- Topic, relationship, data, and source are clearly presented.
- Accurate graph shows a line of best fit.
- Student includes two or more predictions and a summary of results.

**Extra credit**

- Student chooses one relationship where it is expected that the points will fall in a straight line, and another where there is some scatter.
- A geometric pattern is explored such as data on width and area of figures, or number of figures in a sequence and figure perimeter.
- A slider is used in Fathom to fit a curve to nonlinear data.

**LESSON 2.3**

**EXERCISES**

1. $2n; 70$. The diagrams below show the results for one line through four lines. Notice that the number of regions is always twice the number of lines. So, the function rule is $2n$, and 35 concurrent lines divide the plane into 70 regions.

![Diagram showing regions]

<table>
<thead>
<tr>
<th>Lines</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>$n$</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regions</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>$2n$</td>
<td>70</td>
</tr>
</tbody>
</table>

2. $n + 1; 36$. The diagrams below show the results for one parallel line through four parallel lines. Notice that the number of regions is always one more than the number of lines. Therefore, the rule is $n + 1$, and 35 parallel lines divide the plane into 36 regions.

![Diagram showing regions]

<table>
<thead>
<tr>
<th>Lines</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>$n$</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regions</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>$n + 1$</td>
<td>36</td>
</tr>
</tbody>
</table>

3. $n - 3; 32$. From each vertex, you can draw a diagonal to any vertex except the two adjacent vertices and the vertex itself. Therefore, if a polygon has $n$ sides, you can draw $n - 3$ diagonals from each vertex. So, if a polygon has 35 sides, you can draw 32 diagonals.

![Diagram showing diagonals]

<table>
<thead>
<tr>
<th>Sides</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>$n$</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagonals</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>$n - 3$</td>
<td>32</td>
</tr>
</tbody>
</table>

4. $\frac{n(n - 3)}{2}; 560$. For an $n$-sided polygon, you can draw $n - 3$ diagonals from each vertex (see Exercise 3) for a total of $n(n - 3)$ diagonals. However, this counts each diagonal twice. Therefore, the total number of diagonals for an $n$-sided polygon is $\frac{n(n - 3)}{2}$. So, if a polygon has 35 sides, you can draw $\frac{35(35 - 3)}{2}$, or 560, diagonals.

![Diagram showing diagonals]

<table>
<thead>
<tr>
<th>Sides</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>$n$</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagonals</td>
<td>0</td>
<td>2</td>
<td>5</td>
<td>9</td>
<td>14</td>
<td>$\frac{n(n - 1)}{2}$</td>
<td>560</td>
</tr>
</tbody>
</table>

5. $\frac{n(n - 1)}{2}; 595$. This is essentially the same as the handshake problem in the investigation. If there are $n$ points, you need to draw $n - 1$ segments (each point must be connected to each point but itself) for a total of $n(n - 1)$ segments. However, this counts each segment twice, so you must divide by 2. Therefore, for $n$ points, you need to draw...
\[ \frac{n(n-1)}{2} \] segments. So, for 35 points, you need to draw \[ \frac{35(35-1)}{2} \], or 595, segments.

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Points} & 1 & 2 & 3 & 4 & 5 & n \\hline
\text{Segments} & 0 & 1 & 3 & 6 & 10 & \frac{n(n-1)}{2} \\hline
\end{array}
\]

6. \[ \frac{n(n-1)}{2} \], 595. If there are \( n \) lines, then each line intersects each of the other \( n - 1 \) lines for a total of \( n(n - 1) \) intersection points. However, this counts each intersection point twice, so you must divide by 2. Therefore, for \( n \) lines, there are \[ \frac{n(n-1)}{2} \] intersection points. So, for 35 lines, there are \[ \frac{35(35-1)}{2} \], or 595, intersection points.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Lines} & 1 & 2 & 3 & 4 & n \\hline
\text{Intersections} & 0 & 1 & 3 & 6 & \frac{n(n-1)}{2} \\hline
\end{array}
\]

7. Visualize the points in Exercises 5 and 6 as vertices of a polygon, as in Exercise 4. The total number of segments connecting \( n \) random points is the number of diagonals of the \( n \)-sided polygon, \[ \frac{n(n-3)}{2} \], plus the number of sides, \( n \). Thus, the total number of segments connecting \( n \) random points is

\[
\frac{n(n-3)}{2} + \frac{2n}{2} = \frac{n^2 - 3n + 2n}{2} = \frac{n^2 - n}{2} = \frac{n(n-1)}{2}
\]

8. 780. If each house must be connected to each of the other houses by a direct line, then this situation is similar to Exercise 5 (and to the handshake problem). The formula for \( n \) houses is \[ \frac{n(n-1)}{2} \]. So, for 40 houses, 780 lines are needed. It is more practical to have a central hub with a line to each house, so that only 40 lines are needed. The diagrams show the direct-line solution and the practical solution for six houses.

9. 180 games are played. Possible model: First suppose each team plays each of the other teams once. Use points to represent the teams and segments to represent the games. This is the same model used in Exercise 5 (and the handshake problem). If \( n \) teams each play each other once, \( \frac{n(n-1)}{2} \) games are played. If they play each other four times, \[ 4 \cdot \frac{n(n-1)}{2} \], or \( 2n(n-1) \), games are played. For a ten-team league, \( 2(10)(10 - 1) \), or 180, games are played.

10. If \[ \frac{n(n-1)}{2} \] = 66, then \( n(n-1) \) = 132. Find two consecutive integers whose product is equal to 132. Because 12 \( \cdot \) 11 = 132, \( n \) must be 12. Thus, there were 12 people at the party.

11. True

12. True

13. False. An isosceles right triangle has two congruent sides.

14. False. Here, \( \angle AED \) and \( \angle BED \) are not a linear pair.

15. False. They are parallel.

16. True

17. False. A rectangle is a parallelogram with all of its angles congruent.


19. True

20. 5049. In each figure, the number of rows of circles is one more than the figure number, while the number of columns is one less than twice the figure number. Thus, the \( n \)th figure will contain \( (n + 1) \) rows and \( (2n - 1) \) columns of circles, for a total of \( (n + 1)(2n - 1) \) circles. This means that the \( n \)th term in the numbers pattern is \( (n + 1)(2n - 1) \), so the 50th term will be \( (50 + 1)(2 \cdot 50 - 1) = (51)(99) = 5049 \).

**IMPROVING YOUR VISUAL THINKING SKILLS**
**Extension**

Below are several examples of quadratic functions. In each case the constant second difference is twice the coefficient of the squared term.

\[ y = 3x^2 - 10 \]

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-7</td>
<td>2</td>
<td>17</td>
<td>38</td>
<td>65</td>
<td>98</td>
<td>137</td>
</tr>
</tbody>
</table>

\[ y = -x^2 + 2 \]

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>-2</td>
<td>-7</td>
<td>-14</td>
<td>-23</td>
<td>-34</td>
<td>-47</td>
</tr>
</tbody>
</table>

\[ y = 2x^2 - 3x + 1 \]

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>3</td>
<td>10</td>
<td>21</td>
<td>36</td>
<td>55</td>
<td>78</td>
</tr>
</tbody>
</table>

**Lesson 2.4**

**Exercises**

1. inductive; deductive

2. \( m \angle B = 65^\circ \). This problem is solved deductively by subtracting 25° from 90°.

3. Each term includes a figure and a number. Each figure is a regular polygon with one more side than the previous polygon and contains two fewer dots than the number of sides. The numbers are squares of consecutive integers. The next two terms are shown below. This solution involves observing patterns and making generalizations, which is inductive reasoning.

4. \( DG = 258 \text{ cm}; \) deductive. \( TD + DG + GT = 756. \)

Let \( x \) be the length of \( DG \). Then \( x \) is also the length of \( TD \). So, \( x + x + 240 = 2x + 240 = 756. \) Therefore, \( 2x = 516, \) so \( x = 258 \text{ cm}. \) This solution involves deductive reasoning because it uses a sequence of logical statements that are based on agreed-upon assumptions and proven facts.

5. \( m \angle 1 = 20^\circ; m \angle 2 = 32^\circ; m \angle 3 = 37^\circ; m \angle 4 = 27^\circ; m \angle 5 = 64^\circ. \) The sum of these angle measures is 180°. If this pattern continues, the sum of the marked angles in star E will also be 180°. This solution involves inductive reasoning.

6. LNDA is a parallelogram. This conclusion involves deductive reasoning because it is based on agreed-upon assumptions.

7. Possible answer: Use the same figure that appears in Step 2 of the Overlapping Segments Investigation.

![Diagram of quadrilateral ABD with segments AD, BD, and AC labeled.](image)

From the given information, \( AB = CD, \) so by the Addition Property of Equality, \( AB + BC = CD + BC. \) Using Segment Addition gives \( AB + BC = AC \) and \( CD + BC = BD; \) so, by Substitution, \( AC = BD. \) By the Definition of Congruent Segments, \( AC \equiv BD. \)

8. a. \( CD = AB = 3 \)

b. By the Overlapping Segments Property, \( BD = AC = 10. \)

c. By the Overlapping Segments Property, \( AC = BD = 4 + 3 = 7. \)

9. Just over 45°. The smallest possible obtuse angle is just over 90°. So, the smallest possible acute angle formed when an obtuse angle is bisected is just over \( \frac{1}{2} \times 90^\circ, \) or 45°.

10. \( m \angle CPB = 48^\circ; m \angle APD = 17^\circ; m \angle CPB = 62^\circ. \)

Conjecture: If points C and D lie in the interior of \( \angle APB, \) and \( m \angle APC = m \angle DPB, \) then \( m \angle APD = m \angle CPB. \)

11. Possible answer: Refer to the figure for Exercise 10 in the textbook. From the given information, \( m \angle APC = m \angle BPD. \) Add the same measure to both sides to get \( m \angle APC + m \angle CPD = m \angle BPD + m \angle CPD. \) By Angle Addition, \( m \angle APD + m \angle CPD = m \angle APD \) and \( m \angle BPD + m \angle CPD = m \angle CPB. \) By Substitution, \( m \angle APD = m \angle CPB. \)

12. Sample answer: I wanted to buy a CD that cost $15.89. I had $17, and I wasn’t sure it would be enough. The sales tax is 5%. I know that 10% of $16 is $1.60, so 5% of $16 would be half of that, or $0.80. So, if the CD had cost $16, the total price would have been $16.80. Because the actual cost was $15.89, I knew the total price would have to be less than that. So, I figured out that I did have enough money. This is an example of deductive reasoning because it involves a series of logical steps, each of which is based on facts.

13. The pattern cannot be generalized because once the river is straight, it cannot get any shorter.

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14. 900, 1080. Each term is 180 more than previous term.
15. 75, 91. Add 10, then add 11, then add 12, and so on.
16. \( \frac{3}{5}, 12 \). The terms alternate between fractions and integers. The integers are consecutive. The denominators of the fractions are consecutive integers, and each numerator is one less than its denominator.
17. The first term is one line, the second term is two intersecting lines, and the third term is three lines, each of which intersects the other two. So, the fourth term is four lines, each intersecting the other three.

![Diagram of intersecting lines]

18. Each term has one more row and one more column than the previous term. The shading alternates between "top half shaded" and "left half shaded."

![Grid with alternating shaded sections]

19. In the next term, the number of rows will increase by 2 and the number of columns will increase by 1. In each corner, a 3-by-3 square will be shaded.

![Extended grid pattern]

20. Each term contains polygons with one more side than the previous term, so the next term must be hexagons. In each term the midpoints of the outer polygon are connected, and then the midpoints of that polygon are connected to form a third polygon.

![Hexagon with connected midpoints]

21. Sample answer: My friend is on the basketball team. She has taken two tests the day after a game and received A's on both of them. She concluded that she will get an A on any test given the day after a game. This is an incorrect use of inductive reasoning. The fact that the two A tests were taken the day after a game is probably just a coincidence. The A's also involved a lot of work and studying by my friend. Using her reasoning, you would conclude that no matter how much she studies for a test, even if she doesn't study at all, she will get an A. This is probably not true.

22. I (Note: K looks like a kite, but without information about the lengths of the other two sides, you cannot conclude that it is a kite.)
23. M
24. A (Note: O looks like a trapezoid, but without information about whether the sides are parallel, you cannot conclude that it is a trapezoid.)
29. D 30. H
31. I (Note: The segment in J looks like an angle bisector, but without information about whether the angles formed are congruent, you cannot conclude that it is an angle bisector.)

32. W
33. B
34. C
35. K

**IMPROVING YOUR VISUAL THINKING SKILLS**
Each gear will rotate in the opposite direction of the gear to its left. Gear E will rotate counterclockwise.

**EXTENSION**
Discussions will vary.

**LESSON 2.5**

**EXERCISES**
In Exercises 1–5, there is more than one way to find each answer. One possible method is described.

1. By the Vertical Angles Conjecture, \( a = 60^\circ \). By the Linear Pair Conjecture, \( b = 120^\circ \) and \( c = 120^\circ \).
2. By the Linear Pair Conjecture, \( a = 90^\circ \) and \( b = 90^\circ \). By the Vertical Angles Conjecture, \( 40^\circ + c = 90^\circ \), so \( c = 50^\circ \).
3. By the Linear Pair Conjecture, \( 51^\circ + (a + 52^\circ) = 180^\circ \), so \( a = 77^\circ \). By the Vertical Angles Conjecture, \( b = 52^\circ \), \( c = 77^\circ \), and \( d = 51^\circ \).
4. By the Vertical Angles Conjecture, \( a = 60^\circ \). By the Linear Pair Conjecture, \( b = 120^\circ \) and \( c = 120^\circ \). Similar reasoning can be used to find the other angles: \( d = 115^\circ \), \( e = 65^\circ \), \( f = 115^\circ \), \( g = 125^\circ \), \( h = 55^\circ \), \( i = 125^\circ \).

5. By the Linear Pair Conjecture, \( a = 90^\circ \). By the Vertical Angles Conjecture, \( b = 163^\circ \). By the Linear Pair Conjecture, \( c = 17^\circ \). By the Linear Pair Conjecture, \( d = 110^\circ \) and \( e = 70^\circ \).

6. The measures of the linear pair of angles add to 170°. They should add to 180°.

7. The angles must be the same size and their measures must add to 90°, so he should cut the ends at a 45° angle.

8. Greatest: 120°; smallest: 60°. Possible explanation: The tree is perpendicular to the horizontal. The angle of the hill measures 30°. The smaller angle and the angle between the hill and the horizontal form a pair of complementary angles, so the measure of the smaller angle is \( 90^\circ - 30^\circ \), or 60°. The smaller angle and the larger angle form a linear pair, so the measure of the larger angle is \( 180^\circ - 60^\circ \), or 120°.

9. The converse is not true. Possible counterexample:

10. Each must be a right angle.

11. Let \( x \) be the measure of each of the congruent angles. The angles are supplementary, so \( x + x = 180^\circ \). Therefore \( 2x = 180^\circ \), so \( x = 90^\circ \). Thus each angle is a right angle.

12. The ratio is always 1. The ratio does not change as long as the lines don’t coincide. The demonstration may convince students that the Vertical Angles Conjecture is true, but it does not explain why it is true.

13. \( P \)

14. \( \) 15. \( (16, \) 17. \( 18. \) Possible answer: All the cards look exactly as they did, so it must be the 4 of diamonds because it has rotational symmetry, while the others do not.

19. \( \frac{360^\circ}{16} = 22.5^\circ \)

20. \( C_{n}H_{2n} \). The number of hydrogen atoms is always twice the number of carbon atoms. So, if there are \( n \) carbon atoms, there are \( 2n \) hydrogen atoms.

21. \( H \ H \ H \ H \ H \ H \ H \ H \ H \ H \ H \ H \ H \ H \ H \ H \ H \ H \ H \ H \ H \ H \)

22. \( 4n + 6; 806; (n + 1)(n + 2); 40,602 \). Possible method for perimeter: Fill in the table for the fourth rectangle. Notice that the perimeter increases by 4 with each rectangle. Use this pattern to fill in the values for 5 and 6. The rule is in the form \( 4n + c \) for some number \( c \). Using the data for case 1, \( 4(1) + c = 10 \), so \( c = 6 \). Therefore, the rule is \( 4n + 6 \). The perimeter of the 200th rectangle is \( 4(200) + 6 \), or 806.

To find the rule for the number of squares, notice for the first few cases that the number of columns for the \( n \)th rectangle is \( n + 1 \), and the number of rows is one more than that, \( n + 2 \). Thus, the total number of \( 1 \times 1 \) squares is \( (n + 1)(n + 2) \). The number of squares in the 200th rectangle is \( 201 \times 202 = 40,602 \).

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Rectangle} & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
\text{Perimeter of rectangle} & 10 & 14 & 18 & 22 & 26 & 30 & \quad 4n + 6 & \quad 806 \\
\hline
\text{Number of squares} & 6 & 12 & 20 & 30 & 42 & 56 & (n + 1)(n + 2) & 40,602 \\
\hline
\end{array}
\]

23. 3160. Possible method: This is essentially the same as the handshake problem, so use the rule \( \frac{n(n - 1)}{2} \). For 80 students, \( \frac{80(79)}{2} \), or 3160, pieces of string are needed.
160. Possible method: If there are 80 lines, then each intersects 79 other lines, for a total of 80(79) intersections. However, this counts each intersection twice, so there are actually \( \frac{80(79)}{2} \), or 3160, intersections.

25. 760. Possible method: There are 40 people and each shakes hands with 38 other people. However, simply multiplying 40 \cdot 38 counts each handshake twice, so you must divide by 2. Therefore the number of handshakes is \( \frac{40(38)}{2} \), or 760, handshakes.

26. 21; 252. Possible method: 21 diagonals can be drawn from each of the 24 vertices for a total of 21 \cdot 24 diagonals. However, this counts each diagonal twice. So, the actual number of diagonals is \( \frac{24(23)}{2} \), or 252.

27. 35. Possible method: Let \( n \) be the number of diagonals, then \( \frac{n(n-3)}{2} = 560 \). So, \( n(n-3) = 1120 \). You can solve this problem by guessing and checking or by rewriting the equation as \( n^2 - 3n - 1120 = 0 \) and then factoring or using the quadratic formula. The solution is \( n = 35 \), so the polygon has 35 diagonals.

28. \( M \) is the midpoint of \( \overline{AY} \); deductive.

**IMPROVING YOUR ALGEBRA SKILLS**

1. The length of the first segment is \( 3(x - 3) + 20 \).
   The length of the second segment is \( 2(2x - 23) + 30 \).
   Because the segments are the same length, set these expressions equal and solve.
   
   \[
   3(x - 3) + 20 = 2(2x - 23) + 30 \\
   3x - 9 + 20 = 4x - 46 + 30
   \]
   Apply the distributive property.
   
   \[
   3x + 11 = 4x - 16 \\
   11 = x - 16
   \]
   Combine the constant terms.
   
   \[
   11 = x - 16 \\
   x = 27
   \]
   Subtract 3x from both sides.
   
   \[
   x = 27
   \]
   Add 16 to both sides.

2. \[
\begin{align*}
\frac{x + 3}{x + 3} & = \frac{14}{14} \\
\frac{x - 4}{x - 4} & = \frac{11}{11}
\end{align*}
\]

**LESSON 2.6**

**Exercises**

1. By the Alternate Interior Angles Conjecture, \( w = 63^\circ \).

2. By the Corresponding Angles Conjecture, \( x = 90^\circ \).

3. No. By the Linear Pair Conjecture, the angle above the 122° angle has measure 58°. Because the corresponding angles indicated below are not congruent, the lines are not parallel.

4. Extend \( \overline{AN} \) to the left and label a point \( Q \) on the extension. Because \( \overline{AN} \) and \( \overline{FU} \) are parallel, the alternate interior angles \( \angle T \) and \( \angle QAT \) are congruent. Thus, \( m\angle QAT = 57^\circ \). Because \( \overline{AT} \) and \( \overline{NU} \) are parallel, the corresponding angles \( \angle N \) and \( \angle QAT \) are congruent. Thus, \( y = m\angle N = 57^\circ \).

5. Yes. Extend \( \overline{FI} \) to the left and label a point \( G \) on the extension. By the Linear Pair Conjecture, \( m\angle HFG = 65^\circ \). Because the alternate interior angles \( \angle SHF \) and \( \angle HFG \) are congruent, \( \overline{HF} \) and \( \overline{IF} \) are parallel by the Converse of the Parallel Lines Conjecture. Because the corresponding angles \( \angle HFG \) and \( \angle I \) are congruent, the same conjecture tells us that \( \overline{HI} \) and \( \overline{HF} \) are parallel. Because quadrilateral \( FISH \) has two pairs of parallel opposite sides, it is a parallelogram.

6. In the diagram below, some of the angles have been numbered. Because \( m \parallel n \), \( m\angle 1 = 67^\circ \) by the Alternate Interior Angles Conjecture. Thus, \( m\angle 2 \) is also \( 67^\circ \). Because \( \angle 2 \) and \( \angle 3 \) are alternate interior angles, \( m\angle 3 = 67^\circ \). By the Linear Pair Conjecture, \( z = 113^\circ \).

7. \( a = 64^\circ \), \( b = 116^\circ \), \( c = 116^\circ \), \( d = 64^\circ \), \( e = 108^\circ \), \( f = 72^\circ \), \( g = 108^\circ \), \( h = 72^\circ \), \( i = 108^\circ \), \( j = 108^\circ \), \( k = 108^\circ \), \( m = 105^\circ \), \( n = 79^\circ \), \( p = 90^\circ \), \( q = 116^\circ \), \( s = 90^\circ \), and \( t = 119^\circ \). By the Vertical Angles Conjecture, \( d = 64^\circ \) by the Alternate Exterior Angles Conjecture, \( b = 116^\circ \) by the Linear Pair Conjecture, \( c = 116^\circ \) by the Alternate Exterior Angles Conjecture, \( e = 108^\circ \) by the Alternate Interior Angles Conjecture,
12. Explanations will vary. Sample explanation: I used the protractor to make corresponding angles congruent when I drew line PQ.

13. No. Tomorrow could be a holiday. Converse: "If tomorrow is a school day, then yesterday was part of the weekend." The converse is also false.

14. $x = 42^\circ$. Look at the angle that forms a linear pair with the angle with measure $x$ and is on the same side of the transversal. This angle and the angle with measure $4x - 30$ form a pair of corresponding angles, so they are congruent by the Corresponding Angles Conjecture. Therefore, the angle forming a linear pair with the angle with measure $x$ must also have measure $4x - 30$. By the Linear Pair Conjecture, the two angles forming the linear pair are supplementary, giving the equation $x + (4x - 30) = 180$. Solve this equation to find the value of $x$.

$$x + (4x - 30) = 180$$
$$x + 4x - 30 = 180$$ Remove parentheses.
$$5x - 30 = 180$$ Combine like terms.
$$5x = 210$$ Add 30 to both sides.
$$x = 42^\circ$$ Divide both sides by 5.

15. $y = 20^\circ$. First look at the pair of corresponding angles with measures $3x + 16$ and $216 - 2x$. By the Corresponding Angles Conjecture, their measures are equal, which gives the equation $3x + 16 = 216 - 2x$. Solve this equation to find the value of $x$.

$$3x + 16 = 216 - 2x$$ Add 2x to both sides.
$$5x = 200$$ Subtract 16 from both sides.
$$x = 40^\circ$$ Divide both sides by 5.

Now look at the pair of angles with measures $7y - 4$ and $216 - 2x$. These are vertical angles, so their measures are equal, which gives the equation $7y - 4 = 216 - 2x$. Substitute 40 for $x$ and then solve the resulting equation to find the value of $y$.

$$7y - 4 = 216 - 2x$$ Substitute 40 for $x$.
$$7y - 4 = 216 - 2(40)$$ Multiply.
$$7y - 4 = 216 - 80$$ Subtract.
$$7y = 140$$ Add 4 to both sides.
$$y = 20^\circ$$ Divide both sides by 7.
16. No. If $x = 12^\circ$, then $4x - 2(6 - 3x) = 4(12) - 2(6 - 3 \cdot 12) = 48 - 2(6 - 36) = 48 - 2(-30) = 48 + 60 = 108$, so the measure of the lower exterior angle in the figure would be $108^\circ$. Now look at the angle directly above the lower exterior angle. It forms a linear pair with the angle whose measure we have found to be $108^\circ$, so its measure is $180^\circ - 108^\circ = 72^\circ$. However, if the lines were parallel, its measure would have to be $80^\circ$ by the Corresponding Angles Conjecture. Therefore, the lines are not parallel.

17. Isosceles triangles (which include equilateral triangles)

18. Parallelograms that are not rectangles, squares, or rhombuses

19. 18 cm

20. 39°

21. Each of the 84 phones is connected to 83 other phones, for a total of 83(84) lines. However, this counts each line twice, so the actual number of lines is $\frac{83(84)}{2}$, or 3486.

22. 30 squares (one 4-by-4, four 3-by-3, nine 2-by-2, and sixteen 1-by-1)

23. The triangle moved to the left one unit. The new triangle is congruent to the original.

24. The quadrilateral was reflected across both axes, or rotated $180^\circ$ about the origin. The new quadrilateral is congruent to the original.

25. The pentagon was reflected across the line $y = x$. The new pentagon is congruent to the original.

26. (See table at bottom of page.)

**PROJECT**

Project should satisfy the following criteria:

- Correct equations are given for the lines:
  
  $y = - \frac{1}{2}x + 1$, $y = - \frac{2}{5}x + 2$, $y = - \frac{3}{5}x + 3$,
  
  $y = - \frac{4}{5}x + 4$, $y = - \frac{5}{2}x + 5$, $y = - \frac{6}{2}x + 6$, and
  
  $y = -7x + 7$.

- Student creates another line design and states equations that match the lines.

**Extra credit**

Student creates several unique line designs.

**EXTENSIONS**

A. Results will vary.

B. The angle properties of transversals crossing parallel lines are true on a cylinder, because the cylinder can be mapped without distortion onto the plane. To visualize this, draw long parallel lines and a transversal on a sheet of paper and wrap the paper into a cylinder, making the parallel lines become circles or helixes.

---

**Lesson 2.6, Exercise 26**

<table>
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<th>Figure number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>$n$</th>
<th>35</th>
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<tr>
<td>Number of yellow squares</td>
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<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>$n + 1$</td>
<td>36</td>
</tr>
<tr>
<td>Number of blue squares</td>
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<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
<td>13</td>
<td>$2n + 1$</td>
<td>71</td>
</tr>
<tr>
<td>Total number of squares</td>
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<td>8</td>
<td>11</td>
<td>14</td>
<td>17</td>
<td>20</td>
<td>$3n + 2$</td>
<td>107</td>
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</tbody>
</table>
USING YOUR ALGEBRA SKILLS 2

EXERCISES

1. \( m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 0}{12 - 16} = \frac{8}{-4} = -2 \)

2. \( m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - (-4)}{-16 - (-3)} = \frac{12}{-13} \)

3. \( m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1.5 - 8.2}{0.7 - 5.3} = \frac{-9.7}{-4.6} \approx 2.1 \)

4. \( \frac{y - 2}{2 - (-5)} = 3 \)
The slope of the line through \((-5, 2)\) and \((2, y)\) is 3.

\( \frac{y - 2}{7} = 3 \)
Simplify the denominator.

\( y - 2 = 21 \)
Multiply both sides by 7.

\( y = 23 \)
Add 2 to both sides.

5. \( \frac{9 - 2}{7 - x} = \frac{7}{3} \)
The slope of the line through \((x, 2)\) and \((7, 9)\) is \(\frac{7}{3}\).

\( \frac{7 - x}{3} = \frac{7}{3} \)
Simplify the numerator.

\( 7(3) = 7(7 - x) \)
Multiply both sides by \(3(7 - x)\).

\( 21 = 49 - 7x \)
Use the distributive property.

\( -28 = -7x \)
Subtract 49 from both sides.

\( 4 = x \)
Divide both sides by -7.

6. The slope of the line through \((0, 0)\) and \((3, -4)\) is \(-\frac{4}{3}\). One way to locate another point on the line is to start at \((3, -4)\) and move down 4 units and right 3 units. This gives \((6, -8)\). To find other points, you can continue this pattern of moving down 4 units and right 3 units. Possible answers: \((6, -8)\), \((9, -12)\), and \((12, -16)\) (Any correct point will be of the form \((3p, -4p)\).)

7. The speed is the slope of the line. Using the points marked, \( m = \frac{-400 - 200}{6 - 3} = \frac{-600}{3} = -200 \text{ mi/h} \).

8. At 6 m/s, Skater 1 is 4 m/s faster than Skater 2 at 2 m/s.

9. A 100% grade has a slope of 1. It has an inclination of 45°, so you probably could not drive up it. You might be able to walk up it. Grades higher than 100% are possible, but the angle of inclination would be greater than 45°.

10. The slope of the adobe house flat roof is approximately 0. The Connecticut roof is steeper to shed the snow.

CHAPTER 2 REVIEW

EXERCISES

1. Diana is using poor inductive reasoning, but she was probably just joking.

2. Sample situation: One night my sister arrived home 15 minutes after her curfew and did not get in trouble. She concluded that she would never get punished as long as she wasn’t more than 15 minutes late. A few days later, she got home 10 minutes late and was punished. My sister used poor inductive reasoning because she based her conclusion on only one observation.

3. Sample situation: When Leslie found out her party was on the same night as a home football game, she said, “Half the people I invited won’t even show up!” Because she invited 20 people, her brother concluded that only 10 people would show up. On the night of the party, 18 guests arrived. This is incorrect deductive reasoning because it is based on a faulty assumption (that half the people would not show up).

4. 19, −30. Each term is the difference of the two previous terms. The next two terms are 8 − (−11), or 19, and −11 − (19), or −30.

5. S, 36. The pattern alternates between numbers and letters. The letters start with A and skip 2 letters, then 3 letters, then 4 letters, and so on. The numbers are the squares of consecutive integers, starting with 2².

6. 2, 5, 10, 17, 26, 37. Here’s how to calculate the first three terms:

\( 1^2 + 1 = 2 \)

\( 2^2 + 1 = 5 \)

\( 3^2 + 1 = 10 \)

7. 1, 2, 4, 8, 16, 32. Here’s how to calculate the first three terms:

\( 2^1 = 2^0 = 1 \)

\( 2^2 = 2^1 = 2 \)

\( 2^3 = 2^2 = 4 \)

8. Look at the pattern on each face separately. The top face appears to be rotating 90° with each term. On the left face, the shaded square alternates between the first and second row and moves down the columns. On the right face, the figures in the four squares seem to be rotating in a clockwise direction and alternating between a solid triangle and an outlined triangle.

9. The figures alternate between a net for a pyramid and a pyramid. The number of sides in the base increases by 1 with each term. Because the last figure shown is a pyramid with a hexagonal base,
the next figure will be the net for a pyramid with a heptagonal base.

10. 900. Look for a pattern in the sums:

\[ 1 = 1 = 1^2 \quad \text{First 1 odd number} \]
\[ 1 + 3 = 4 = 2^2 \quad \text{First 2 odd numbers} \]
\[ 1 + 3 + 5 = 9 = 3^2 \quad \text{First 3 odd numbers} \]
\[ 1 + 3 + 5 + 7 = 16 = 4^2 \quad \text{First 4 odd numbers} \]
\[ 1 + 3 + 5 + 7 + 9 = 25 = 5^2 \quad \text{First 5 odd numbers} \]

Based on this pattern, the sum of the first 30 odd whole numbers is 30^2, or 900.

11. 930. Look for a pattern in the sums:

\[ 2 = 1(2) \quad \text{First 1 even number} \]
\[ 2 + 4 = 6 = 2(3) \quad \text{First 2 even numbers} \]
\[ 2 + 4 + 6 = 12 = 3(4) \quad \text{First 3 even numbers} \]
\[ 2 + 4 + 6 + 8 = 20 = 4(5) \quad \text{First 4 even numbers} \]
\[ 2 + 4 + 6 + 8 + 10 = 30 = 5(6) \quad \text{First 5 even numbers} \]

Based on this pattern, the sum of the first 30 even whole numbers is 30(31), or 930.

12. \(-3n + 5; -55\). Possible method: The difference between \(f(n)\) values is always \(-3\), so the rule is in the form \(-3n + c\) for some number \(c\). Using the first term, \(-3(1) + c = 2\), so \(c = 5\). Therefore, the \(n\)th term is \(-3n + 5\) and the 20th term is \(-3(20) + 5 = -55\).

13. \[ \frac{n(n+1)}{2}, \quad 210 \]

14. \(n^2; 900\). Possible method: The number of blocks in a stack \(n\) blocks high is the sum of the first \(n\) odd whole numbers. From Exercise 10, the sum is \(n^2\). A stack 30 blocks high would require 30\(^2\), or 900, blocks.

15. 5050. Possible method: The number of bricks in stacks like the one shown is always a triangular number, so it’s determined by the formula \(\frac{n(n+1)}{2}\). A stack 100 bricks high would have \(\frac{100(101)}{2}\), or 5050, bricks.

16. \[ \frac{n(n-1)}{2} = 741. \text{ Therefore, } n = 39. \]

17. \[ \frac{n(n-1)}{2} = 2926. \text{ Therefore, } n = 77. \]

18. 52. These diagrams show that for an \(n\)-sided polygon, you can draw \(n - 3\) diagonals from each vertex, dividing the interior into \(n - 2\) regions. So, in a 54-sided polygon, the diagonals would divide the interior into 52 regions.

19. a. Sample answer: \(\angle AFE\) and \(\angle CFG\) (or \(\angle CFH\))

b. Sample answer: \(\angle AFE\) and \(\angle CFE\)

c. Sample answer: \(\angle CFE\) and \(\angle DGF\) (or \(\angle DGE\))

d. Either of these pairs: \(\angle AFG\) and \(\angle DGF\), or \(\angle CFG\) and \(\angle BGF\)

20. \(\angle AFE \cong \angle CFG\) Vertical Angles Conjecture

\(\angle AFE \cong \angle BGE\) Corresponding Angles Conjecture

\(\angle AFE \cong \angle DGH\) Alternate Exterior Angles Conjecture

21. True. Converse: If the polygons have the same number of sides, they are congruent. The converse is false.

Possible counterexample:

22. Draw a ray and use a protractor to draw a 56° angle at the endpoint. Mark a point 7 cm from the endpoint, and use the protractor to draw another 56° angle to create congruent corresponding angles. Extend the side of each angle and mark a point 4.5 cm from the angle vertex. Connect these points. To check that the opposite sides are congruent, measure the corresponding angles and alternate interior angles shown to confirm that they measure 56°.

23. \(\overline{PV} \parallel \overline{RX}\) and \(\overline{SU} \parallel \overline{UX}\). Possible explanation: Look at points \(P, Q, R, S,\) and \(X\). Each of these points is the vertex of four angles, one of which has a given angle measure. Using the Vertical Angles Conjecture and the Linear Pair Conjecture, find the measures of the other three angles at each of these vertices. At \(P\) and \(R\), the angle measures are 132° and 48° (two angles with each measure at each vertex), while at \(S\) and \(X\), the angle measures are 134° and 46°. By
looking at the measures of corresponding angles and applying the Converse of the Parallel Lines Conjecture, \( \overline{PV} \parallel \overline{RX} \) and \( \overline{SU} \parallel \overline{VX} \).

24. Possible answer: Because the two lines cut by the transversal are parallel, the measure of the bisected angle must be 50° by the Alternate Interior Angles Conjecture. Then each half of the bisected angle must measure 25°. The angle bisector is also part of a transversal, so, again by the Alternate Interior Angles Conjecture, the measure of the other acute angle in the triangle must also be 25°. However, this angle and the 165° angle are a linear pair, so the sum of their measures must be 180°; but 165° + 25° = 190°, not 180°.

25. \( a = 38° \), \( b = 38° \), \( c = 142° \), \( d = 38° \), \( e = 50° \), \( f = 65° \), \( g = 106° \), and \( h = 74° \). \( a = 38° \) by the Linear Pair Conjecture, \( b = 38° \) by the Alternate Interior Angles Conjecture, \( c = 142° \) because it forms a linear pair with the alternate interior angle of \( b \), \( d = 38° \) by the Alternate Exterior Angles Conjecture, \( f = 65° \) because it is half of the angle corresponding to the 130° angle, \( e = 50° \) by the Linear Pair Conjecture, \( g = 106° \) by the Corresponding Angles Conjecture, and \( h = 74° \) by the Linear Pair Conjecture.

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**CHAPTER 3**

**LESSON 3.1**

**EXERCISES**

1. To duplicate \( \overline{AB} \), draw a ray that is clearly longer than \( \overline{AB} \). Label the endpoint \( A \). Open your compass to the length of \( \overline{AB} \) in the book. Then, without changing the compass opening, place the sharp end of your compass on point \( A \) of your ray and draw an arc that intersects the ray. Label the point of intersection \( B \). Duplicate \( \overline{CD} \) and \( \overline{EF} \) in the same way.

2. Duplicate segment \( \overline{AB} \) as described in the solution to Exercise 1. Then duplicate segment \( \overline{CD} \) so that the left endpoint is the right endpoint of the previous segment.

3. Draw a long ray and copy \( \overline{AB} \).

4. Follow these steps to duplicate each angle:

   **Step 1** Draw a ray.

   **Step 2** Go back to the angle in the book. Open your compass so that it reaches from the vertex of that angle to the arc.

   **Step 3** Without changing the opening of your compass, place the sharp end on the endpoint of the ray you drew, and draw a large arc.

   **Step 4** Go back to the angle in the book. Notice that the arc intersects the angle in two points. Open your compass so that it reaches from one intersection point to the other.

   **Step 5** Go back to your drawing. Without adjusting the compass, place the sharp end on the point where the arc intersects the ray, and draw a small arc that intersects the large arc.